Applied Probability Modeling of Data Networks

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Collected network data exhibit properties inconsistent with traditional queueing models:

- self-similarity (ss) & long-range dependence (LRD) of various transmission rates:

  packet counts/time,
  www bits/time

- heavy tails

  file sizes,
  transmission rates,
  transmission durations,
  CPU job completion times,
  call lengths

Origins: Bellcore study in early 90's revealed *packet counts per unit time exhibit self similarity and long range dependence*.

Research Goals:

- Understand origins and effects of long-range dependence and self-similarity

- Understand some connections between $\{$ SS & LRD $\}$ and heavy tails.

- Begin to understand the effect of network protocols and architecture.

- Say something useful for the purposes of capacity planning. (Ambitious!)

The infinite node Poisson model:

Infinitely many potential users connected to single server which processes work at constant rate r. At a Poisson time point, some user begins transmitting work to the server at <u>constant</u> (ugh) rate. The length of the transmission is random with heavy tailed distribution.

Features:

- Fairly flexible.

- Offers successful explanation of LRD being caused by heavy tailed job sizes.

- Predicts traffic aggregated over users and accumulated over time $[0, T]$ is approximated by either FBM (Gaussian) or stable Lévy motion (heavy tailed).

• The model does not fit the data sets we examined all that well.

$\rightarrow$ Constant input rates clearly wrong.

$\rightarrow$ No hope the model can successfully match fine time scale behavior observed below, say, 100 milliseconds.

BROAD ISSUES (BI's):

BI 1. Problem of time scales: Can Applied Math, Applied Prob & Statistics make contributions to data network analysis and planning in *internet time*. Maybe not but at least we can help cause paradigm shifts with explanations which may lag behind developments.

BI 2. Insider vs Outsider: Do you

(a) Analyze data that is already available, say on the web (eg, ITA) or that you have bootlegged by hook or by crook. This is often what academics (including me) have done.

Note, for example, the data we analyzed is NOT very suitable for testing the Poisson assumption.

or

(b) Design a network experiment to get the data you want. This typically requires co-operative net administrators and hardware + software expertise. It may require you to go beyond your LAN to something of the scale of World Net, UUNet etc.

BI 3. How can you discern the influence of network architectures and protocols? How do you model something like TCP?

BI 4. The internet was designed to be robust and scalable and there is less emphasis on the word

optimal.

Can you get a reasonably accurate model which encourages analysis beyond simulation and experimentation? (Ambitious!)

BI 4. A broad variety of techniques useful:

Applied Probability & Statistics:

- Stochastic Processes: FBM, Lévy,
- Poisson point process theory,
- weak convergence,
- heavy tailed analysis,
- long-range dependence,
- self-similarity & multifractality,
- extreme value theory,
- estimation methods such as maximum likelihood, graphical techniques,
- time series analysis,
- queueing theory.

Applied Math:

- wavelets (seem to be the right tool for examining phenomena on different time scales),
- numerical methods,
- design and implementation of simulation tools,
- computing.

BI 5. Black box vs structural modeling: Black box modeling provides a class of models with enough parameters to fit a broad variety of data sets. EG: ARMA(p,q) time series models

$$X_n = \sum_{i=1}^{p} \phi_i X_{n-i} + \sum_{i=0}^{q} \theta_i Z_{n-i},$$

where one specifies p, q, ϕ's and θ's to yield a model matching the L_2 sample moments of the data. Traditionally $\{Z_n\}$ is white noise; in a heavy tailed context $\{Z_n\}$ iid and heavy tailed.

Problems

- Do not get good fits for heavy tailed data. **The only heavy tailed data that ARMA fit are simulated data from the ARMA model.**
- Even if ARMA fit acceptable, it cannot provide fundamental insights since physics and structure ignored.

- Just fitting the data may not be so useful in a rapidly changing world: the next data set generated from similar mechanisms may be quite different.

Implies:

$\rightarrow$ More of an emphasis on structural modeling which incorporates features of the network,

$\rightarrow$ Try to use the detailed information in packet "headers".

BI 6. Two data sets are ancient (internet standards). Still useful? BU and UCB from 1996.

Some Technical Points

Tech Pt 1: Identify **Poisson time points** and validate the choice statistically.

Quick & dirty (Q&D): Check if interpoint distances are iid (sample acf almost 0) and exponentially distributed (qq-plots).

Q&D Rules of Thumb:

- Behavior of lots of humans acting independently is often well modelled by a Poisson process.
- Starting times of machine triggered downloads cannot be modelled as Poisson process.

EXAMPLE: UCB data; http sessions via modem.

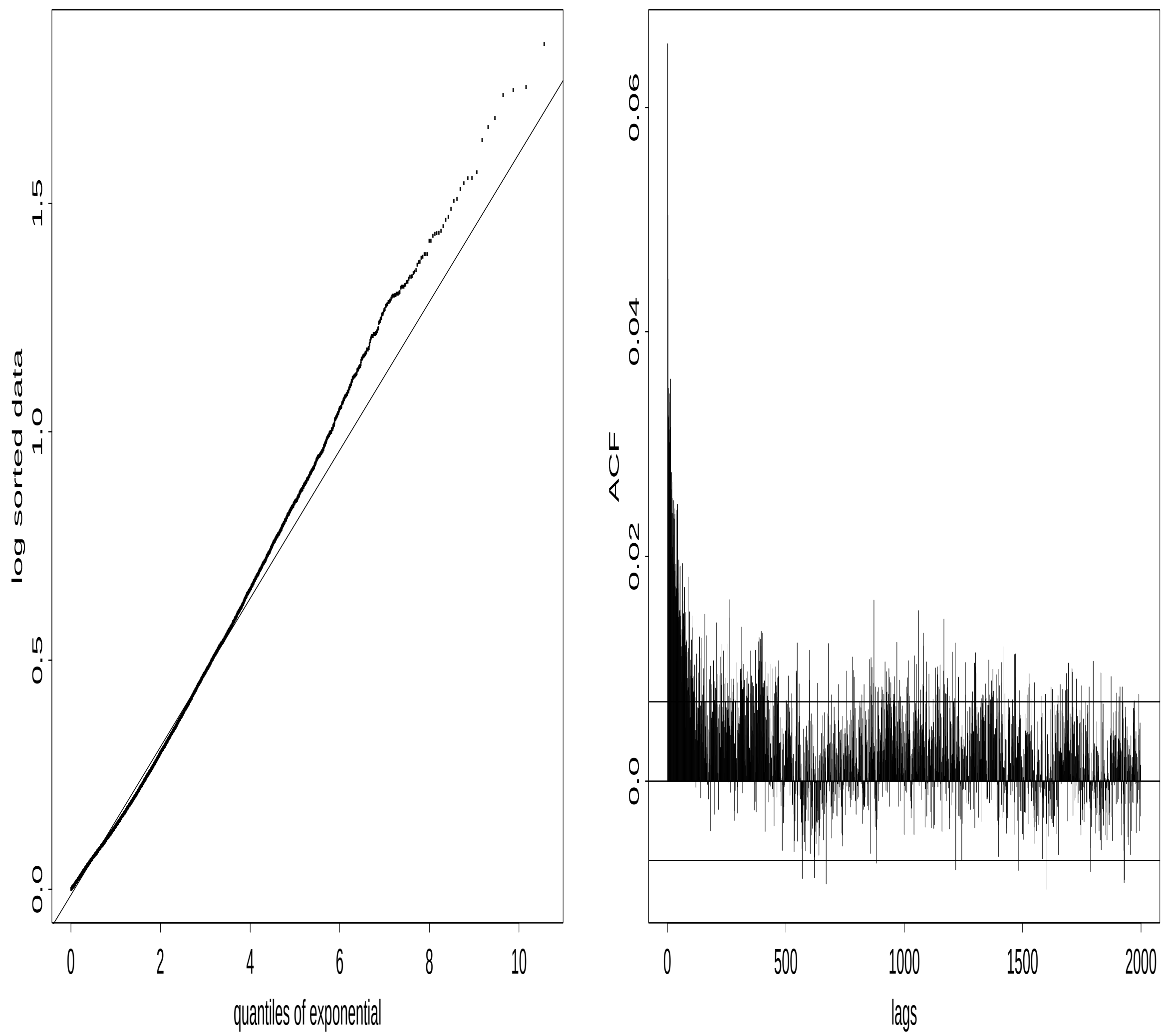


Figure: UCB inter-arrival times of requests: *left*) qqplot against exponential distribution, *right*) autocorrelation function of interarrival times.

Tech Pt 2: **Heavy tails**: A rv X has a heavy (right) tail if

$$P[X > x] \sim x^{-\alpha} L(x), \quad x \to \infty.$$

Cases:

(i) Very heavy: $0 < \alpha < 1$. Mean & variance infinite (Eg: BU file sizes)

(ii) Heavy: $1 < \alpha < 2$. Usual case where mean finite but variance is infinite.

(iii) Heavy with finite variance: $\alpha > 2$. Typical of financial data.

EXAMPLE: BU data: File sizes downloaded in a web session.

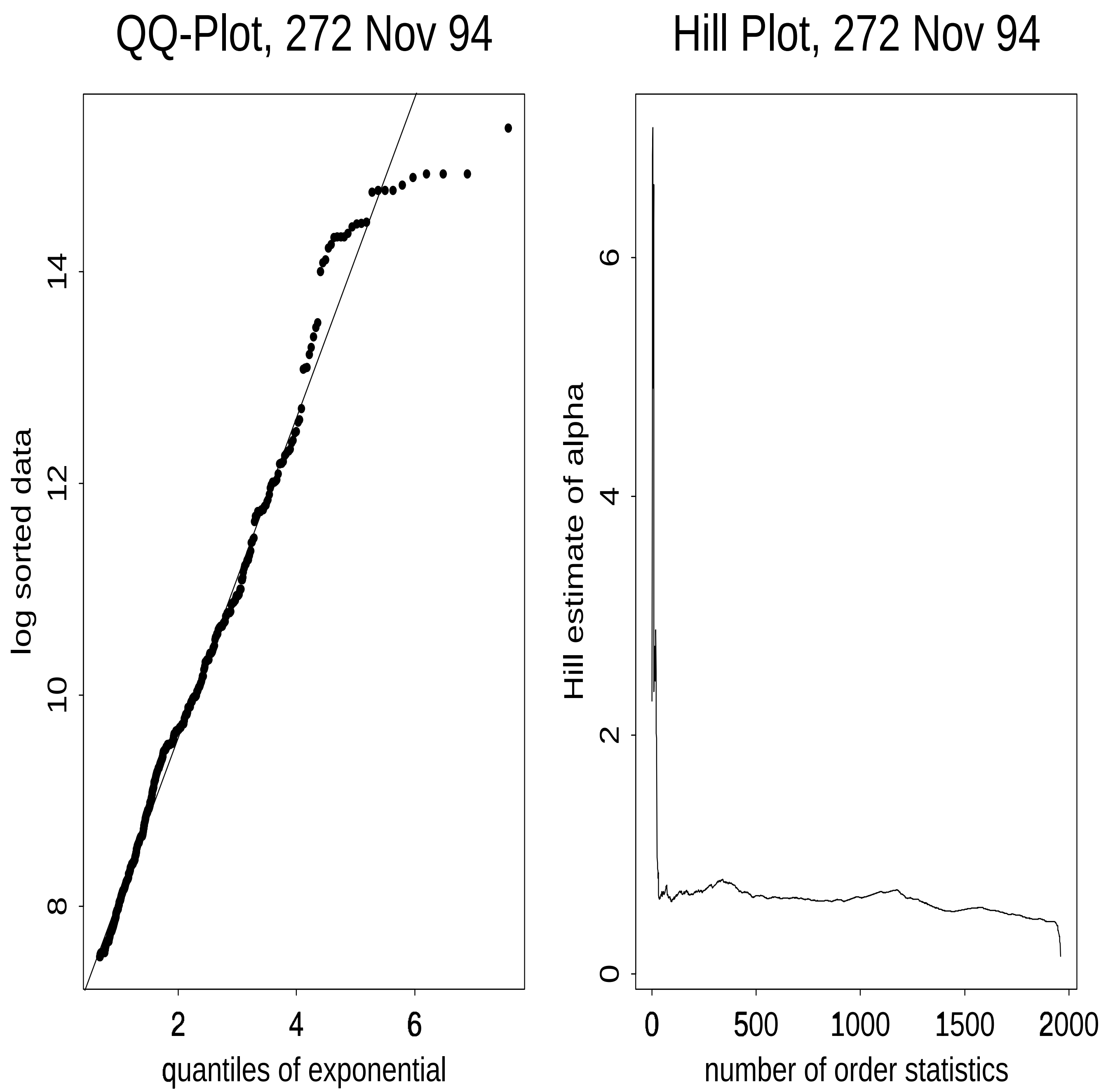


Figure: Sizes of www downloads; BU experiment: QQ-plot and Hill plot.

For many purposes, do not need to know the whole distribution but just the tail.

Techniques for deciding when a heavy tailed model is appropriate:

- Hill plots (modified MLE) and refinements (smoothing the Hill plot, plotting on different scales)
- qq-plots of the log(order statistics) vs exponential quantiles.
- Mean residual life plots.
- Extreme value theory techniques such as

  $\rightarrow$ DeHaan moment estimator of $1/\alpha$

  $\rightarrow$ Peaks over threshold modeling leading to fitting of a generalized Pareto distribution by method of MLE. Splus software available (free) from McNeil. However, estimates sensitive to choice of threshold.

Data set	γ (=1/α)
simM/G/∞	$-.13 \pm .03$
BUburst 10s	$-.36 \pm .13$
BUburst 1s	$.17 \pm .03$
UCB 10s	$.05 \pm .18$
UCB syn 10s	$-.60 \pm .14$
Munich lo TX	$.09 \pm .04$
Munich lo RX	$-.03 \pm .05$
Munich hi .1s	$.17 \pm .12$
Munich hi .01s	$.10 \pm .03$
Ericsson	$-.47 \pm .09$
Eri syn 1s	$-.31 \pm .12$

Table: Point estimates for traffic rate tail $\pm$ "standard deviation".

Tech Pt 3. **Stable distributions**: A particular class of heavy tailed distributions: Theory predicts that under certain assumptions, cumulative traffic in $[0, T]$, aggregated over all users, can be approximated by a stable Lévy motion (stable marginals).

The density of a stable distribution does not exist but can estimate parameters (shape, location, symmetry) using numerical maximum likelihood using John Nolan software.

Tech Pt 4. How to check for **independence**.

Q&D method 1: check if sample correlation function

$$\hat{\rho}(h) = \frac{\sum_{i=1}^{n-h}(X_i - \bar{X})(X_{i+h} - \bar{X})}{\sum_{i=1}^{n}(X_i - \bar{X})^2}, \quad h = 1, 2, \ldots,$$

is **close** to identically 0. How to put meaning to phrase **close** to 0? If

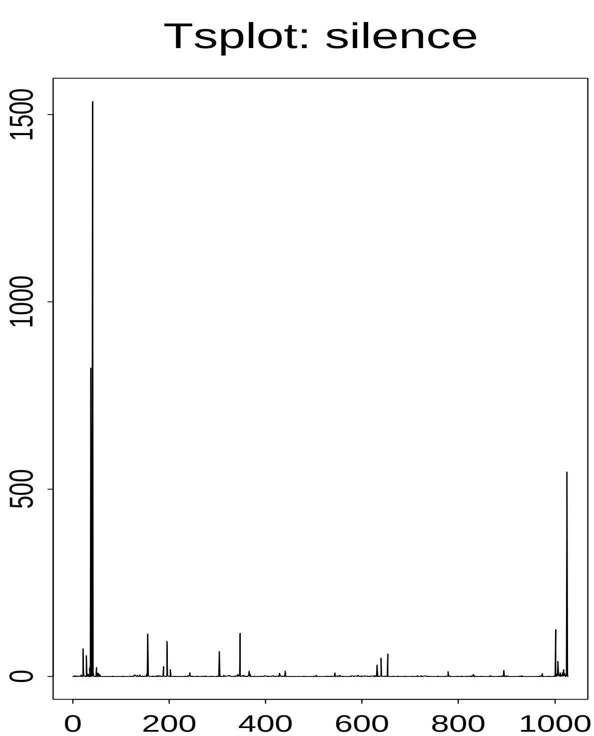
(a) If finite variances, Bartlett's formula provides asymptotic normal theory.

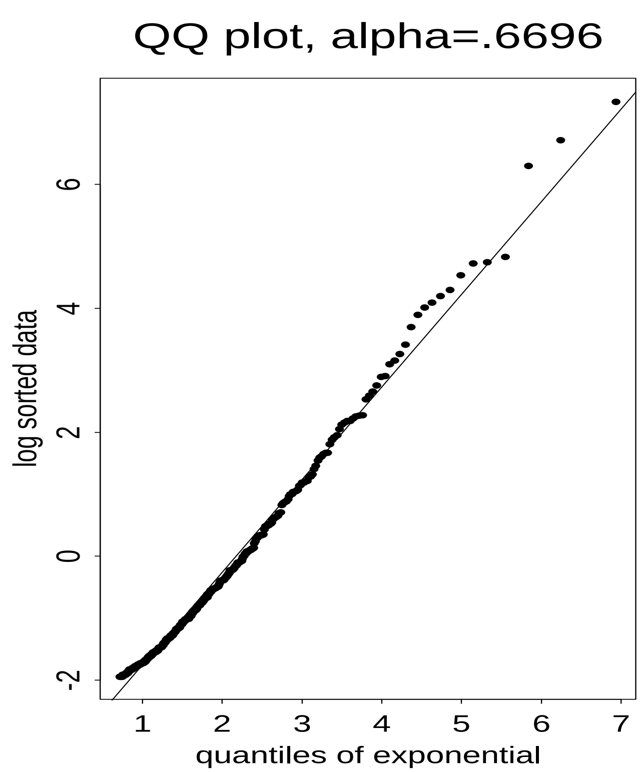
(b) If heavy tailed, Davis and Resnick formula provides asymptotic distributions for $\hat{\rho}(h)$.

Q&D method 2: If data heavy tailed, take a function of the data (say the log) to get lighter tail and test. (But this may obscure the importance of large values.)

Q&D method 3: Subset method. Split data into (say) 2 subsets. Plot acf of each half separately. If iid, pics should look same.

EXAMPLE: *Silence*: 1026 times between transmission of packets at a terminal.





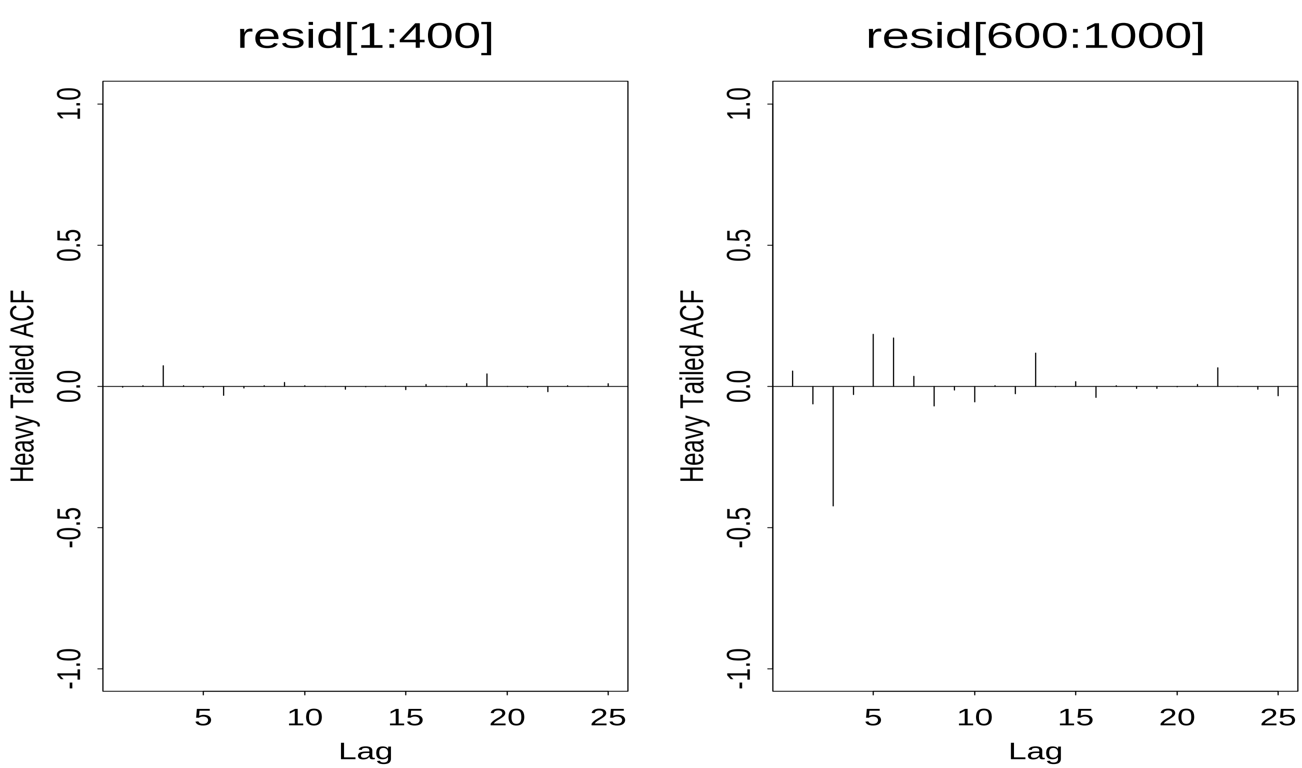
Try to fit black box time series model to the data. Best available techniques suggest AR(9)

$$X_n = \sum_{i=1}^{9} \phi_i X_{n-i} + Z_n,$$
$$n = 9, \ldots, 1029 - 9, \{Z_n\} \text{ iid.}$$

Estimate the coefficients and a goodness of fit test for the AR model is:

Q: Are the residuals iid?

A: Nope.



Tech Pt 5. Is the data **stationary**? Usually not and there are, for example, diurnal cycles.

Q&D Coping Method: Take a slab of the copius data which looks stationary.

- Rule of thumb: don't take more than 4 hours.
- Should we try to model the cycles?

Tech Pt 5. **Long range dependence**. A stationary L_2 sequence $\{\xi_n, n \geq 1\}$ has long-range dependence if

$$\mathrm{Cov}(\xi_n, \xi_{n+h}) \sim h^{-\beta} L(h), \quad h \to \infty$$

for $0 < \beta < 1$.

How to test? Q&D method: Sample acf should not $\to 0$ quickly.

EXAMPLE: Ericsson–a days worth of http file transfers to and from a corporate www Ericsson server; # bytes sent/second.

Tech Pt 6. **Hurst exponent and self-similarity.** A process $\{X(t), t \geq 0\}$ is H-self-similar (H-ss) if for any $a > 0$ (distributional scaling property)

$$X(a\cdot) =^d c^H X(\cdot).$$

Paradigm: FBM is a Gaussian process which is H-ss and having covariance

$$\text{Cov}(B_H(t), B_H(s)) = c(t^{2H} + s^{2H} - |t - s|^{2H}).$$

$\rightarrow$ Estimates of H given by **wavelet** regression method developed by Abry and Veitch.

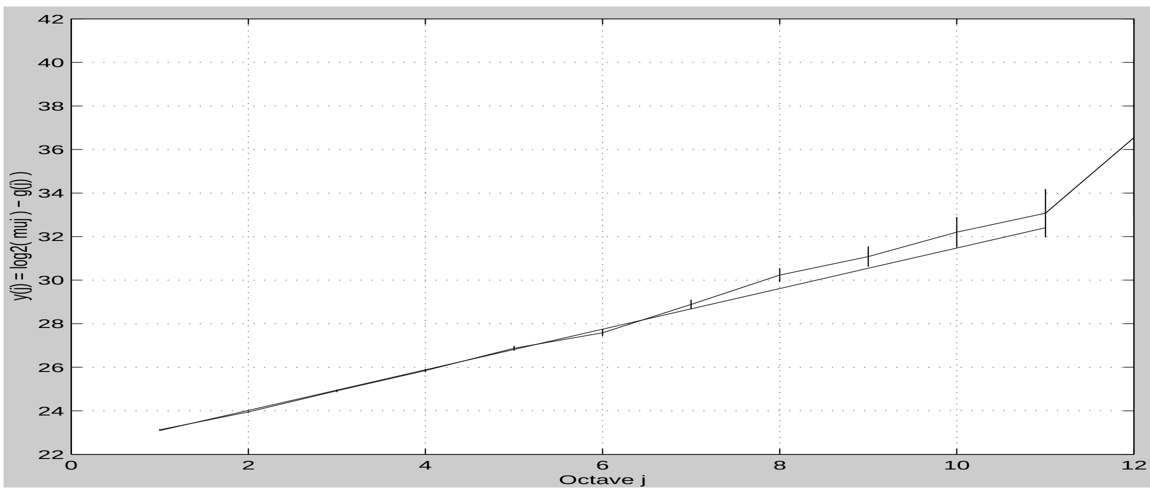
$\rightarrow$ Connection between LRD and H-ss: If $\{\xi_n\}$ has LRD, then block averages of ξ's approximately increments of FBM (ie, FGN).

$\rightarrow$ Theory predicts that under certain assumptions, cumulative traffic in $[0, T]$, aggregated over all users, can be approximated by a FBM.

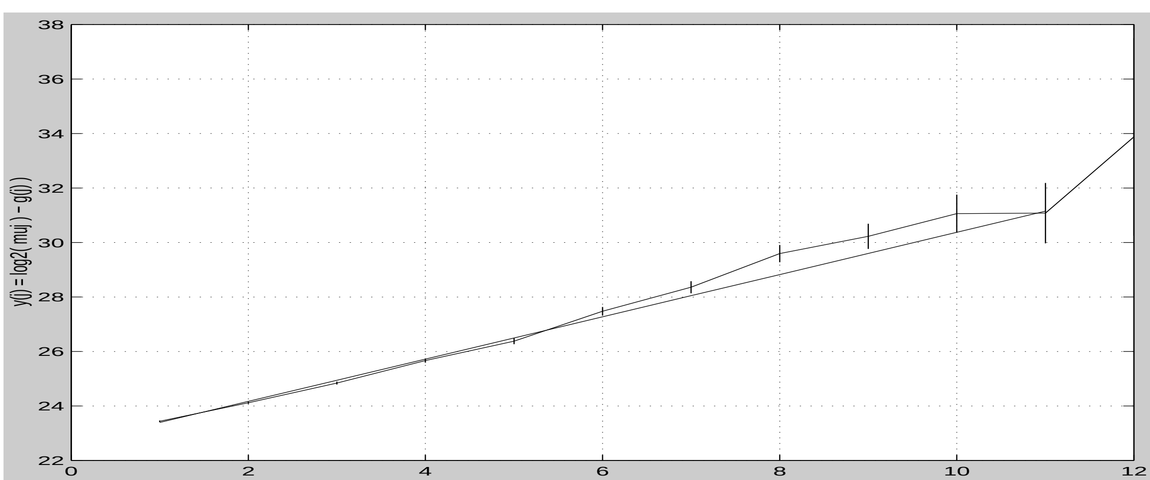
EXAMPLE: "A picture is worth a thousand words." (Willinger)

EXAMPLE:

Munich lo, RX: Wavelet regression estimation of the Hurst exponent.



Munich lo, TX: Wavelet regression estimation of the Hurst exponent.



Tech Pt 7. **Hölder exponents.** For a process $\{A(t), t \geq 0\}$ the Hölder exponent at t (if it exists) is ($\tau \to 0$)

$$E\Big(A(t+\tau) - A(t)\Big)^2 = c\tau^{2H_o(t)} + o(\tau^{2H_o(t)}),$$

$\to$ For FBM: $H \equiv H_o(t)$.

$\to$ If empirical estimates reject constancy of $H_o(\cdot)$ then have evidence against FBM approximation.

$\to$ Estimate $H_o(t)$ using a ratio estimator of Istas based on quadratic variation. This relies on limit theorem for Gaussian processes:

$$n^{2H_o(0)-1}\frac{1}{2}\sum_{k=1}^{n}\Big(X(\frac{k}{n}) - X(\frac{k-1}{n})\Big)^2 \to^{a.s.} c.$$

→ Empirical evidence exists that on small time scales ($<$ 100 milliseconds) $H_o(t)$ is not constant and it is at these time scales where one must seek the influence of protocols and network architecture.

→ Do we need **multifractal processes?**

Data set	$\hat{H}$	$\hat{H}_o$
simM/G/∞	$.90 \pm .01$	.88
BUburst 10s	$.89 \pm .02$	.73
BUburst 1s	$.81 \pm .01$	.87
UCB 10s	$.58 \pm .03$	.65
UCB syn 10s	$.95 \pm .07$	1.36
Munich lo TX	$.89 \pm .01$	.86
Munich lo RX	$.97 \pm .01$	.85
Munich hi .1s	$1.02 \pm .03$	.66
Munich hi .01s	$.1.03 \pm .04$	.56
Ericsson	$.88 \pm .02$	1.21
Eri syn 1s	$1.48 \pm .02$	1.51