System Dynamics Examples

Note Title 2/24/2005

Spreadsheet SysDyn01.xls has three dynamical models.

Model 1

Inventory Management

Inventory managers try to maintain inventory by ordering more production to replenish stocks.

Inventory in period t: I_t

Production order in period t: P_t

Production lead time: L

Outstanding production orders: O_t

$$O_t = \sum_{j=1}^{L-1} P_{t-j}$$

Inventory target: S

Production rule: $P_t = S - (I_t + O_t)$

Demand in period t : D_t

Inventory Dynamics:

$$I_{t+1} = I_t + P_{t-L} - D_t$$

Now suppose the marketing department also tries to influence inventory.

When inventory gets high:

offer price discounts

When inventory gets low

charge price premium.

Prices affect demand.

Let π_t denote the price in period t
Let $\bar{\pi}$ denote the nominal price.

$$D_{t+1} = D_t \left(\frac{\bar{\pi}}{\pi_t}\right)^{\alpha}$$

where α reflects how sensitive the market is to the relative price.

Marketing rule:

$$\pi_{t+1} = \bar{\pi}\left(\frac{S}{I_t}\right)^{\beta}$$

where β is a policy parameter: how quickly to adjust price.

Observe that the marketers are somewhat stupid: they are ignoring the outstanding productions orders.

When MBA students play the famous "Beer Game" they are being led into similar behavior.

Given initial values of $I_0, P_{-1}, P_{-2}, \dots, P_{-6}$ D_0, the dynamical model is well specified.

The behavior of the system of interest:

- long run average inventory
- long run average demand rate
- stability of the system

as a function of α and β.

One lesson is that myopic rule based behavior easily leads to cyclic outcomes.

Model 2 Satellite Battery Design

Solar Panel on satellite generates electricity.

Satellite orbits earth: cycles in and out of sunlight.

Solar panel charges battery during sunlight phase.

Battery powers satellite in both phases.

Problem: battery efficiency (its ability to accept a charge) varies with temperature.

Battery temperature cycles with sunlight phases.

- warms up when in sunlight
- cools down in shadow

Because of inefficiency of cold batteries the design team is considering adding a heater to maintain the temp. of the battery.

The design decision will have to look at many tradeoffs.
- What are they? heater mass, heater power consumption, size of solar panel required with + without heater.

etc.

Let's just study the temperature dynamics.

Position of satellite in orbit : D_t
measured in degrees
$0 \leq D_t \leq 360$

Sunlight status : $S_t \in \{true, false\}$

$$S_t = \begin{cases} 1 & \text{if } D_t \geq 180 \\ -1 & \text{otherwise} \end{cases}$$

Temperature of satellite : T_t

$$T_{t+1} = T_t + \gamma S_t$$

where γ is the rate of heat accumulation.
(a non-linear model is possible here)

Temperature of Battery: B_t

Heater status : $H_t \in \{\underset{\text{off}}{0}, \underset{\text{on}}{1}\}$

Battery temperature dynamics

$$B_{t+1} = B_t \left(\frac{T_t}{B_t}\right)^{\alpha} + \beta H_t$$

where α is rate of temperature adjustment towards ambient (satellite) temperature.

and β is heat output of heater, when on.

To complete the model, we need a control policy for the heater.

$$H_t = \begin{cases} 1 & \text{if } B_t < \underline{B} \\ 0 & \text{otherwise} \end{cases}$$

The output of this model includes the time profile of battery temperature, B_t from which you can compute efficiency and the time profile of heater usage from which you can compute power consumption.

Model 3

A company offers two levels of service to its subscribers

Regular : L
Premium : H

The premium service subscribers pay a price premium π to belong.

The population of subscribers is divided into two types of individuals:

High : individuals who like fast service and are willing to pay for it.
Low : individuals who like fast service but are not willing to pay (much) for it.

At any point in time, the population is divided into the following groups:

LL : low-type subscribers in low-type service

LH : low-type subscribers in high-type service

HL : high-type in low service

HH : " " " high "

Let N_{gt} denote the number of subscribers in group g at time t

$g \in G = \{LL, LH, HL, HH\}$.

Let $G_L = \{LL, LH\}$

$G_H = \{HL, HH\}$

$\sum_{g \in G_L} N_{gt} = N_L$ number of subscribers <u>un</u>willing to pay much for service

$\sum_{g \in G_H} N_{gt} = N_H$ total high service customers.

The price premium can be changed over time.

Let π_t denote the price premium in period t.

The traffic intensity of the two services is given by

$$\rho_{Lt} = \frac{N_{LLt} + N_{HLt}}{C_L}$$

$$\rho_{Ht} = \frac{N_{LHt} + N_{HHt}}{C_H}$$

where C_L is the server capacity of the regular service and C_H is the server capacity of the premium service.

The service levels achieved by the two services depend on the traffic intensities.

The expected customer queue lengths are

$$Q_{Lt} = \frac{\rho_{Lt}}{1-\rho_{Lt}}$$

$$Q_{Ht} = \frac{\rho_{Ht}}{1-\rho_{Ht}}$$

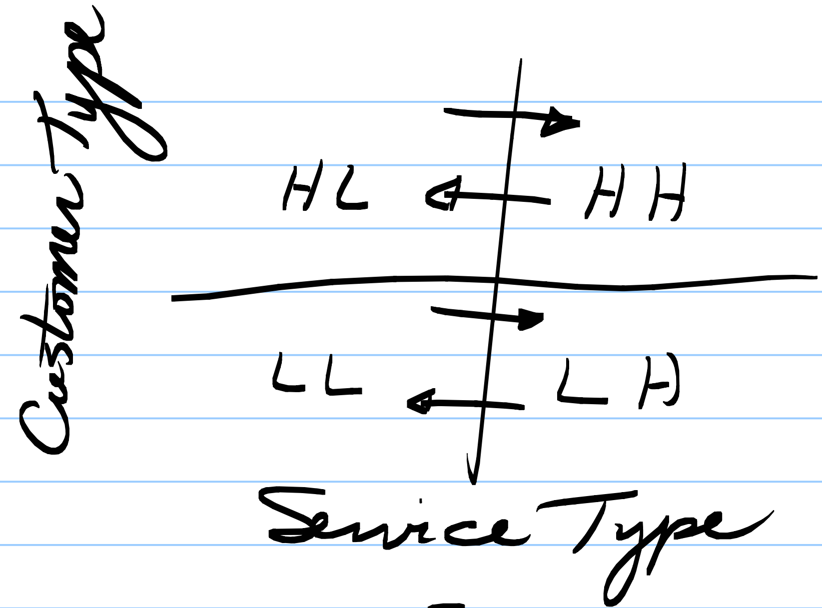
Subscribers compare the price premium to the queue length advantage

$$R_t = \frac{\pi_t}{Q_{Lt} - Q_{Ht}}$$

Each population has a threshold value for this parameter that would induce them to switch from one service to the other. For high-type customers this threshold is R_H. For low type customers this threshold is R_L

Let $W_{gg',t}$ indicate the fraction of subscribers in population who would like to switch to a different service indicated by g' in period t

There are four possibilities:



$$W_{LL\to LH,t} = \begin{cases} 0 & \text{if } R_t < R_L \\ 1-e^{R_t-R_L} & \end{cases}$$

$$W_{LH\to LL,t} = \begin{cases} 0 & \text{if } R_t > R_L \\ 1-e^{R_L-R_t} & \end{cases}$$

Similarly for the other rates.

However, because of contracts not everyone switches at once. Let $S_{gg',t}$ denote the actual number of subscribers switching from group g to group g' in period t.

$$S_{LL \to LH, t} = \omega_{LL \to LH, t} \cdot N_{LL\,t} \cdot \gamma$$

where γ is the maximum rate at which subscribers can change their service level.

γ would be fixed by the contract duration.

Even for a fixed price premium

$\pi_t = \overline{\pi}$, the model has interesting behavior.

We see, for example, that the system approaches a steady state in which all the high type customers gravitate to the high service. Some low type customers stay in the high service category. The premium service is ~~does~~ in fact give better service.

It is likely that if we departed

from a fixed price policy and let π_t depend on, say, $N_{HHt} + N_{LHt}$, then we would introduce de-stabilizing behavior.