1 Test Problems

Our test problems are drawn from numerous sources in the literature. In order to come up with a rich set of problems, we also generated additional test problems using the previously published ones as starting points. We briefly describe our problems below.

Electricity planning – This problem concerns the optimal allocation of limited capacity to different power terminals. The first stage decisions are the capacities assigned to each power terminal, subject to certain resource limitations. The second stage variables are how much power is supplied from each terminal to each demand location. The objective is to maximize the total expected profit, which can include the capacity installation cost, production cost, transportation cost and revenue from sales. The problem can be stated as

$$\min \quad \sum_{i \in \mathcal{P}} c_i \ x_i + Q(x)$$

$$\text{subject to} \quad \sum_{i \in \mathcal{P}} a_{ik} \ x_i \le r_k \qquad \qquad \text{for all } k \in \mathcal{R},$$

$$Q(x,\xi) = \min \quad \sum_{i \in \mathcal{P}} \sum_{j \in \mathcal{C}} p_{ij} \ y_{ij}$$

$$\text{subject to} \quad \sum_{j \in \mathcal{C}} y_{ij} \le x_i \qquad \qquad \text{for all } i \in \mathcal{P}$$

$$\sum_{i \in \mathcal{P}} y_{ij} \le \xi_j \qquad \qquad \text{for all } j \in \mathcal{C},$$

where we omit the nonnegativity constraints for brevity. Each unit of capacity allocated to terminal i uses a_{ik} units of resource k, whose availability is limited to r_k. ξ_j is the random demand at demand location j. A problem with the same structure is given in Louveaux & Smeers (1988).

Bi-weekly fleet size planning – Here, we consider the fleet-sizing problem faced by a freight carrier over a two-week horizon. The loads for the first week are known. The decisions are for the first week are the number of vehicles available at each terminal and the number of vehicles moving between each origin and destination. The decisions for the second week are similar, but the vehicle supply at each terminal is determined by the decisions of the first week. If the vehicle supply of a location at the end of the second week is different than what it was at the beginning of the first week, then a penalty is incurred. We formulate the problem as

$$\min \quad \sum_{i \in \mathcal{L}} v_i \ z_i + \sum_{i,j \in \mathcal{L}} (r_{ij} \ x_{ij} + \tilde{r}_{ij} \ \tilde{x}_{ij}) + Q(z,\tilde{z})$$

$$\text{subject to} \quad \sum_{j \in \mathcal{L}} (x_{ij} + \tilde{x}_{ij}) - z_i = 0 \qquad \qquad \text{for all } i \in \mathcal{L}$$

$$\sum_{i \in \mathcal{L}} (x_{ij} + \tilde{x}_{ij}) - \tilde{z}_j = 0 \qquad \qquad \text{for all } j \in \mathcal{L}$$

$$x_{ij} \le u_{ij} \qquad \qquad \text{for all } i,j \in \mathcal{L},$$

$$Q(z,\tilde{z},\xi) = \min \quad \sum_{i,j \in \mathcal{L}} (r_{ij} \ y_{ij} + \tilde{r}_{ij} \ \tilde{y}_{ij}) + \sum_{j \in \mathcal{L}} p_j \ w_j$$

$$\text{subject to} \quad \sum_{j \in \mathcal{L}} (y_{ij} + \tilde{y}_{ij}) = \tilde{z}_i \qquad \qquad \text{for all } i \in \mathcal{L}$$

$$\sum_{i \in \mathcal{L}} (y_{ij} + \tilde{y}_{ij}) + w_j - \tilde{w}_j = z_j \qquad \qquad \text{for all } j \in \mathcal{L}$$

$$y_{ij} \le \xi_{ij} \qquad \qquad \text{for all } i,j \in \mathcal{L}.$$

x_{ij} and $\tilde{x}_{ij}$ are respectively the number of vehicles moving empty and loaded from terminal i to j during the first week. z_i is the number of vehicles deployed at terminal i. The number of vehicles at terminal i at the beginning of the second week is $\tilde{z}_i$. If the vehicle supply at location j at the end of the second week is below z_j, then this imbalance is penalized by p_j per unit shortage. ξ_{ij} is the random number of loads that need to be carried from terminal i to j in the second stage.

Weekly fleet size planning – This problem is taken from Mak, Morton & Wood (1999), and it is very similar to the previous one, except that we try to impose a one-week cycle instead of a two-week cycle. We also assume that there are multiple vehicle types, which we represent by the set $\mathcal{K}$. The only decisions in the first stage are the number of vehicles of each type deployed at each terminal. The problem is

$$
\begin{aligned}
\min \quad & \sum_{i \in \mathcal{L}} \sum_{k \in \mathcal{K}} v_i^k \, z_i^k + Q(z) \\
\text{subject to} \quad & \sum_{i \in \mathcal{L}} z_i^k \leq f^k && \text{for all } k \in \mathcal{K},
\end{aligned}
$$

$$
\begin{aligned}
Q(z, \xi) = \min \quad & \sum_{i,j \in \mathcal{L}} \sum_{k \in \mathcal{K}} \left(r_{ij}^k \, y_{ij}^k + \tilde{r}_{ij}^k \, \tilde{y}_{ij}^k \right) + \sum_{j \in \mathcal{L}} \sum_{k \in \mathcal{K}} p_j^k \, w_j^k \\
\text{subject to} \quad & \sum_{j \in \mathcal{L}} \left(y_{ij}^k + \tilde{y}_{ij}^k \right) = z_i^k && \text{for all } i \in \mathcal{L}, \; k \in \mathcal{K} \\
& \sum_{i \in \mathcal{L}} \left(y_{ij}^k + \tilde{y}_{ij}^k \right) + w_j^k - \tilde{w}_j^k = z_j^k && \text{for all } j \in \mathcal{L}, \; k \in \mathcal{K} \\
& \sum_{k \in \mathcal{K}} y_{ij}^k \leq \xi_{ij} && \text{for all } i, j \in \mathcal{L},
\end{aligned}
$$

where f^k is the maximum number of vehicles of type k that we can use.

Product distribution – This problem models the operations of a manufacturing company that ships its production from numerous production plants to numerous warehouses, before seeing the realization of the random customer demands. We use $\mathcal{P}$, $\mathcal{W}$ and $\mathcal{C}$ to denote the set of production plants, warehouses and customer locations, respectively. The problem can be stated as

$$
\begin{aligned}
\min \quad & \sum_{i \in \mathcal{P}} \sum_{j \in \mathcal{W}} c_{ij} \, x_{ij} + Q(z) \\
\text{subject to} \quad & \sum_{j \in \mathcal{P}} x_{ij} = p_i && \text{for all } i \in \mathcal{P} \\
& \sum_{i \in \mathcal{P}} x_{ij} - z_j = 0 && \text{for all } j \in \mathcal{W},
\end{aligned}
$$

$$
\begin{aligned}
Q(z, \xi) = \min \quad & \sum_{j \in \mathcal{W}} \sum_{k \in \mathcal{C}} d_{ij} \, y_{jk} - \sum_{k \in \mathcal{C}} r_k \, w_k \\
\text{subject to} \quad & \sum_{k \in \mathcal{C}} y_{jk} \leq z_j && \text{for all } j \in \mathcal{W} \\
& \sum_{j \in \mathcal{W}} y_{jk} \leq \xi_k && \text{for all } k \in \mathcal{C},
\end{aligned}
$$

where ξ_k is the random demand at customer location k.

Telecommunications network design – This problem, due to Sen, Doverspike & Cosares (1994), arises in the context of allocating limited capacity to different links in a telecommunications network.

Demand for communication between different nodes in the network randomly arrives into the system. If there is a path from the origin to the destination node of the demand with sufficient capacity, then the demand is satisfied. Otherwise, the demand is lost. The objective is to minimize the expected number of lost demands. We let $\mathcal{G} = (\mathcal{N}, \mathcal{A})$ be the underlying network, and $\mathcal{P}_{ij}$ be the set of paths connecting node i to node j. The problem is

$$\min \quad Q(z)$$
$$\text{subject to} \quad \sum_{a \in \mathcal{A}} z_a \leq c,$$

$$Q(z, \xi) = \min \quad \sum_{i,j \in \mathcal{N}} w_{ij}$$
$$\text{subject to} \quad \sum_{i,j \in \mathcal{N}} \sum_{p \in \mathcal{P}_{ij}} \delta_{ap} \, y_p \leq z_a \qquad \text{for all } a \in \mathcal{A}$$
$$\sum_{p \in \mathcal{P}_{ij}} y_p + w_{ij} = \xi_{ij} \qquad \text{for all } i, j \in \mathcal{N},$$

where δ_{ap} is a binary variable that takes value 1 if and only if arc a is in path p. ξ_{ij} is the random demand for communication from node i to j.

References

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