Data Network Models of Burstiness

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1. Introduction

Measurements on data networks exhibit features surprising by the standards of classical queueing and telephone network models.

Measurements often consist of data giving bit-rate or packet rates: Select window resolution of (for example)

- 10 seconds
- 1 second
- 10 milliseconds
- 1 millisecond
- ...
- ...

and count number of bits or packets in adjacent windows or slots.

Significant examples:

- Willinger et al. (1997)
- Duffy et al. (1993)
- Leland et al. (1993)
- Willinger et al. (1995)

1.1. BUT:

1. Theoretical attempts to create models to explain the empirical observations concentrate on large time scales and cumulative traffic over large time intervals.

   See

   - Taqqu et al. (1997)
   - Konstantopoulos and Lin (1998)
   - Heath et al. (1998)
   - Levy and Taqqu (2000)
   - Mikosch et al. (2002)
   - Maulik and Resnick (2003)
   - Kaj and Taqqu (2004)

2. For such models, it is difficult to find agreement with many existing data sets (Guerin et al. (2003)).

2. Stylized facts

Many network data sets exhibit distinctive properties, which in analogy with empirical finance, we term *stylized facts*.

2.1. First list:

1. Heavy tails abound for such things as
   - file sizes,
   - transmission rates,
   - transmission durations.

   (See Arlitt and Williamson (1996), Leland et al. (1994), Maulik et al. (2002), Resnick (2003), Resnick and Rootzén (2000), Willinger (1998), Willinger and Paxson (1998), Willinger et al. (1998).)
2. The number of bits or packets per slot exhibits long range dependence across time slots (eg, Leland et al. (1993), Willinger et al. (1995). There is also a perception of self-similarity as the width of the time slot varies across a range of time scales exceeding a typical round trip time.
3. Network traffic is bursty with rare but influential periods of very high transmission rates punctuating typical periods of modest activity.

3. Burstiness

Burstiness, a somewhat vague concept, is an important feature of traffic:

- Introduces sudden peak loads to the network.
- Important for design
- Important for quality of service.

Attempts to understand this phenomenon empirically:

- α/β decomposition of users (Sarvotham et al. (2005)) where
  - α-users transmit large files at very high rate and
  - β-users transmit the rest.
- Alternative language creates a dichotomy between mice and elephants (Azzouna et al. (2004)) depending on whether a file is typical or very large.

3.1. Burstiness: Stylized facts

Some stylized facts suggested by the stimulating empirical study (Sarvotham et al. (2005)) include:

- Large files over fast links contribute to α-traffic. The α-component consitutes a small fraction of total workload but is responsible for burstiness. Often a single dominent high-rate connection causes a burst.
- Most of the dependence structure across time slots is carried by the β-traffic. The long range dependence structure of the β-traffic approximates that of the complete traffic.
- The quantity of traffic in a time window is distributionally approximated by the normal distribution when there is high levels of aggregation across users and heavy loading. Furthermore, β-traffic is much more likely to appear Gaussian than α-traffic.

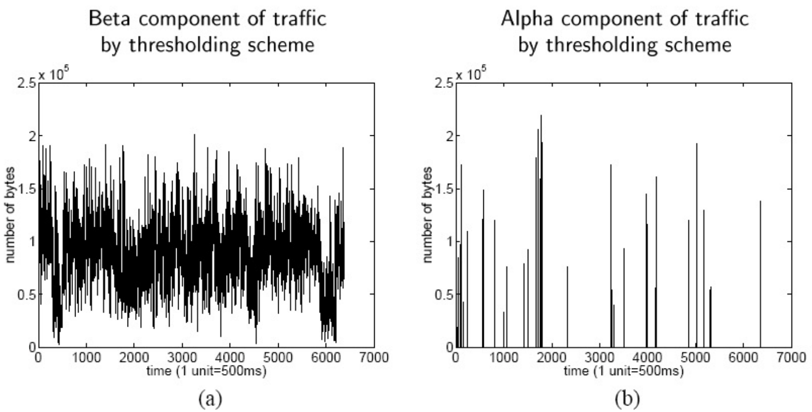


Figure 1: (a) Bytes-per-time arrival process at 500ms aggregation level for the Beta component of the traffic using thresholding scheme on Auck-2. Note its Gaussian character. (b) Similar Alpha component. Note its bursty character. (Quoted from Sarvotham et al. (2005) without permission.)

4. Approach to modeling

Suppose sessions characterized by

- Initiation times $\{\Gamma_k\}$ where

$$\{\Gamma_k\} \sim \text{ Poisson, rate } \lambda.$$

- Mark of Γ_k:

$$(F_k, L_k, R_k) = (\text{file, duration, rate }).$$

- Look at $A(k\delta, (k+1)\delta]$, the work inputted in $(k\delta, (k+1)\delta]$.
- Approximation as $\delta \to 0$? Will need $\lambda = \lambda(\delta) \uparrow \infty$ (a la heavy traffic limit theorems).
- Compute dependence measure across different slots.

Will this explain the stylized facts?

4.1. Difficulties

1. What is a reasonable assumption for the joint distribution of (F, L, R). Statistical studies somewhat inconclusive but point toward the following possibilities:

    - F, R independent?
    - L, R independent?
    - Mixture of the 2 cases?
    - Some asymptotic form of independence?
        - Undoubtedly, no pair of (F, L, R) is truly independent.
        - Statistical evidence points to asymptotic independence of some sort.
        - BUT: Is asymptotic independence worth the cost in complexity? Temporarily, at least, we decided no.

2. The concept of burstiness has no precise definition.

Big Bill Broonzey on deciding that a song was a folk song:

I never heard
a horse sing it.

4.2. Assessing dependence structure of (F, L, R).

Consider

$$\begin{aligned} F &= \text{file size}, \\ L &= \text{duration of transmission}, \\ R &= \text{throughput} \ = F/L. \end{aligned}$$

All three, are seen empirically to be heavy tailed:

$$\begin{aligned} P[F > x] &= x^{-\alpha_F} L_F(x) \\ P[L > x] &= x^{-\alpha_L} L(x) \\ P[R > x] &= x^{-\alpha_R} L_R(x). \end{aligned}$$

Two studies:

- BU
- UNC

What is the dependence structure of (F, R, L)?

Since $F = LR$, the tail parameters $(\alpha_F, \alpha_R, \alpha_L)$ cannot be arbitrary.

Note for BU measurements, we have the following empirical estimates:

α	$\hat{\alpha}_F$	$\hat{\alpha}_R$	$\hat{\alpha}_L$
estimated value	1.15	1.13	1.4

Two theoretical possibilities:

- If (L, R) have a joint distribution with multivariate regularly varying tail but are NOT asymptotically independent then (Maulik et al. (2002))

$$\hat{\alpha}_F = \frac{\hat{\alpha}_L \hat{\alpha}_R}{\hat{\alpha}_L + \hat{\alpha}_R} = .625 \neq 1.15.$$

- If (L, R) obey a form (not the EVT version) of asymptotic independence, (Heffernan and Resnick (2005), Maulik et al. (2002))

$$tP[\left(L, \frac{R}{b(t)}\right) \in \cdot] \xrightarrow{v} F_L(\cdot) \times \alpha x^{-\alpha_R - 1} dx$$

then

$$\alpha_F = \alpha_R \bigwedge \alpha_L$$

and in our example

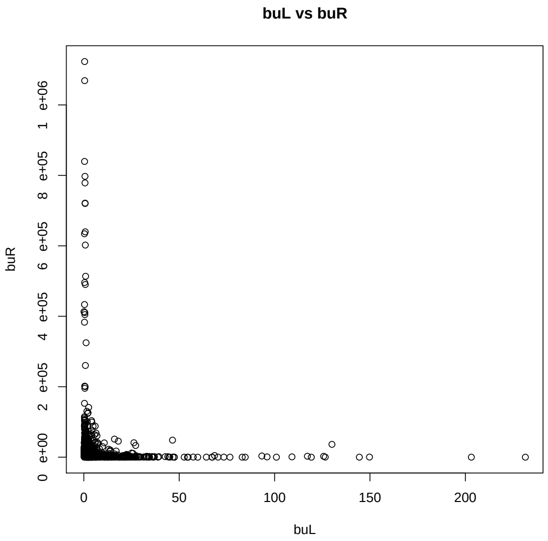
$$1.15 \approx 1.13 \bigwedge 1.4.$$

For two examples

- BU: Evidence seems to support some form of independence for (R, L).
- UNC: Conclusions from Campos et al. (2005);
  - Large values of F tend to be independent of large values of R.
  - Large files do not seem to receive any special consideration when rates are assigned.
  - A form of asymptotic independence for F, R seems appropriate.
- Not a consistent pattern (visible to naked eye).

BuL vs BuR Scatterplot:

Data processed from the original 1995 Boston University data; 4161 file sizes (F) and download times (L) noted and transmission rates (R) inferred. The data consists of bivariate pairs (R,L).



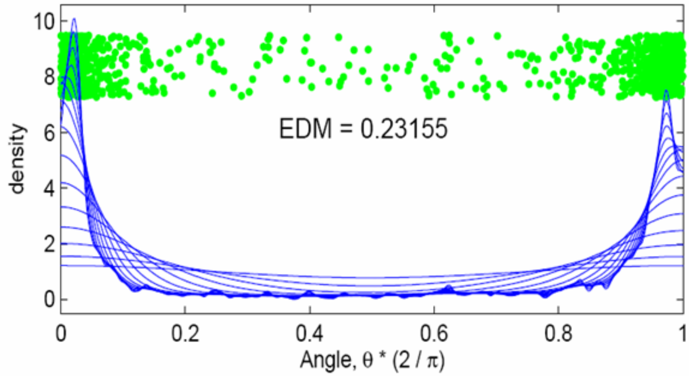
Rank method–UNC

Steps:

- Transform (F,R) data using rank method.
- Convert to polar coordinates.
- Keep 2000 pairs with biggest radius vector.
- Compute density estimate for angular measure S.

Plot: Density estimates with various amounts of smoothing+jitter plot (green) of angles.

Full disclosure: These types of plots can be rather sensitive to choice of threshold.



EDM = 0.23155
density
Angle, θ * (2 / π)

5. Model & Results

Assume:

- Sessions begin: $\{\Gamma_k\}$, homogeneous Poisson rate λ.
- For the k-th session, independently attach iid marks (F_k, R_k, L_k); F, R independent, heavy tailed; $F = LR$,

$$F \sim G(x) \quad R \sim F_R(x).$$

- $1 < \alpha_F, \alpha_L, \alpha_R < 2$; finite means, infinite 2nd moments.
- Distribution tail of L given by

$$\bar{F}_L(l) \sim \mathbb{E}\left(\frac{1}{R}\right)^{\alpha_F} \bar{G}(l),$$

  provided assume (Breiman (1965))

$$\mathbb{E}\left[\frac{1}{R}\right]^{\alpha_F+\eta} < \infty,$$

  for some $\eta > 0$.

- Time slots $(k\delta, (k+1)\delta], k = 0, \pm 1, \pm 2, \ldots\}$.

- Limiting procedure shrinks the observation windows ($\delta \to 0$). To get limit, increase the arrival rate $\lambda = \lambda(\delta)$ of sessions.
- Heavy traffic limit theorem philosophy; move through a family of models indexed by δ as $\delta \downarrow 0$. Choice of λ:

$$\lambda(\delta) = \frac{1}{\delta \bar{F}_R(\delta^{-1})}.$$

- Since $1 < \alpha_R < 2$, this choice of λ guarantees

$$\lambda(\delta) = \frac{1}{\delta^{\alpha_R+1} L_R(\delta^{-1})} \to \infty \quad \text{and} \quad \delta\lambda(\delta) = \frac{1}{\delta^{\alpha_R} L_R(\delta^{-1})} \to \infty.$$

- Seek limit behavior of

$$\mathbf{A}(\delta) := \{A(k\delta, (k+1)\delta], -\infty < k < \infty\}$$

where

$$A(k\delta, (k+1)\delta] = \text{ work inputted in time } (k\delta, (k+1)\delta],$$

as

  - $\delta \to 0$, OR
  - δ is fixed and we study $\mathrm{Cov}(A(0, \delta], A(k\delta, (k+1)\delta])$ as $k \to \infty$ to seek LRD.

5.1. Basic Technique

1. The counting function of the points $\{(\Gamma_k, R_k, L_k, F_k)\}$

$$N = \sum_k \epsilon_{(\Gamma_k, R_k, L_k, F_k)} \tag{1}$$

on $\mathbb{R} \times [0, \infty)^3$ is *Poisson random measure* with mean measure

$$\lambda ds P[(R_1, L_1, F_1) \in (dr, dl, du)] =: \mu^{\#}(ds, dr, dl, du) \tag{2}$$

remembering $F_1 \perp\!\!\!\perp R_1$ and $L_1 = F_1/R_1$.

2. For a region $A \subset \mathbb{R} \times [0, \infty)^3$ with $\mu^{\#}(A) < \infty$,

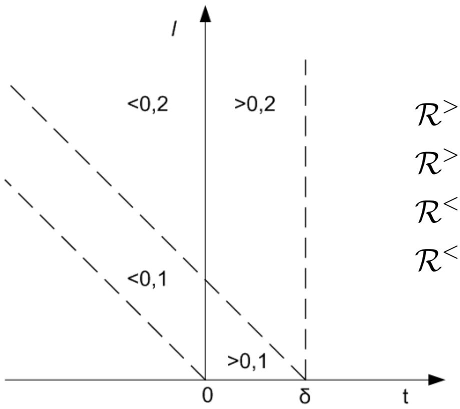
$$N\Big|_A(\cdot) = \sum_{i=1}^{P^A} \epsilon_{\boldsymbol{\xi}_i}(\cdot)$$

where

$$P^A \sim \mathrm{PO}(\mu^{\#}(A))$$

$$\{\boldsymbol{\xi}_i\} \sim \text{iid}\left(\frac{\mu^{\#}\Big|_A(\cdot)}{\mu^{\#}(A)}\right).$$

5.2. Limits for $A(0,\delta]$.



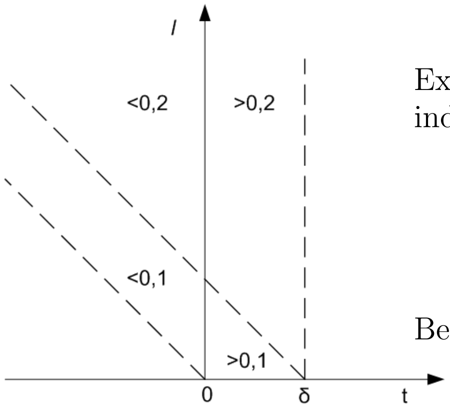
$$\mathcal{R}^{>0,1} = \{(s,r,l,u) : 0 < s \le \delta,\ 0 < s+l \le \delta\},$$
$$\mathcal{R}^{>0,2} = \{(s,r,l,u) : 0 < s \le \delta,\ s+l > \delta\},$$
$$\mathcal{R}^{<0,1} = \{(s,r,l,u) : s < 0,\ 0 < s+l \le \delta\},$$
$$\mathcal{R}^{<0,2} = \{(s,r,l,u) : s < 0,\ s+l > \delta\}.$$

$$A^{>0,1}(\delta) = \sum_k R_k L_k 1_{[(\Gamma_k, R_k, L_k, F_k) \in \mathcal{R}^{>0,1})]},$$

$$A^{>0,2}(\delta) = \sum_k R_k (\delta - \Gamma_k) 1_{[(\Gamma_k, R_k, L_k, F_k) \in \mathcal{R}^{>0,2})]},$$

$$A^{<0,1}(\delta) = \sum_k R_k (L_k + \Gamma_k) 1_{[(\Gamma_k, R_k, L_k, F_k) \in \mathcal{R}^{<0,1})]},$$

$$A^{<0,2}(\delta) = \sum_k R_k \delta 1_{[(\Gamma_k, R_k, L_k, F_k) \in \mathcal{R}^{<0,2})]}.$$



Express $A(0,\delta) =: A(\delta)$ as the sum of 4 independent contributions:

$$A(\delta) = A^{>0,1}(\delta) + A^{>0,2}(\delta) + A^{<0,1}(\delta) + A^{<0,2}(\delta).$$

Behavior of the rv's $A^{(\cdot)}(\delta)$ is as follows:

- $A^{<0,1}(\delta) \stackrel{d}{=} A^{>0,2}(\delta)$;
- $A^{<0,2}(\delta)$ does not converge weakly without scaling and with centering and scaling converges to a Gaussian rv;
- $A^{>0,2}(\delta)$, suitably centered, converges weakly to an infinitely divisible rv with finite variance and whose Lévy measure has a regularly varying tail with index $-(\alpha_F + \alpha_R)$, where $\alpha_F + \alpha_R > 2$.
- $A^{>0,1}(\delta)$ converges in distribution to a compound poisson random variable;

5.3. Method for region $\mathcal{R}^{>0,2}$.

Each of the 4 terms is a compound Poisson sum and therefore it is possible to compute the chf. For example,

$$\mathbb{E}\Big(e^{i\theta A^{>0,2}(\delta)}\Big) = \exp\Big\{\int_{s=0}^{\infty}(e^{i\theta s}-1)\nu_\delta^{>0,2}(ds)\Big\}. \tag{3}$$

where

$$\nu_\delta^{>0,2}(ds) =(\nu_\delta^{>0,2})'(s)ds = \bar{G}(s)\Big(\int_{r=s}^{\infty} r^{-1}\mu_\delta(dr)\Big)ds$$

$$\mu_\delta(dr) :=\frac{F_R(\delta^{-1}dr)}{\bar{F}_R(\delta^{-1})}.$$

Proceed:

- Let $\delta \to 0$.
- For region $> 0, 2$,

$$\nu_\delta^{>0,2} \to \nu_0^{>0,2},$$

  where $\nu_0^{>0,2}$ is a Lévy measure.
- Center to get $A^{>0,2}(0,\delta] - m^{>0,2}(\delta)$ converges to id distribution.

5.4. Method for region $\mathcal{R}^{<0,2}$.

We conclude

$$\mathbb{E}e^{i\theta A^{<0,2}(\delta)} = \exp\{\int_0^\infty (e^{i\theta r} - 1)\nu_\delta^{<0,2}(dr)\}$$

where

$$\nu_\delta^{<0,2}(dr) = \mathbb{E}(F) r^{-1} \bar{G}_0(r)\mu_\delta(dr), \qquad \bar{G}_0(r) = \int_r^\infty \frac{\bar{G}(v)}{\mathbb{E}(F)} dv$$

$$\mu_\delta(dr) = \frac{F_R(\delta^{-1}dr)}{\bar{F}_R(\delta^{-1})} \qquad \bar{G}_0 \in RV_{-(\alpha_F - 1)}$$

Proceed:

- Let $\delta \to 0$.
- For region $\mathcal{R}^{<0,2}$,

  $$\nu_\delta^{<0,2} \to \nu_0^{<0,2},$$

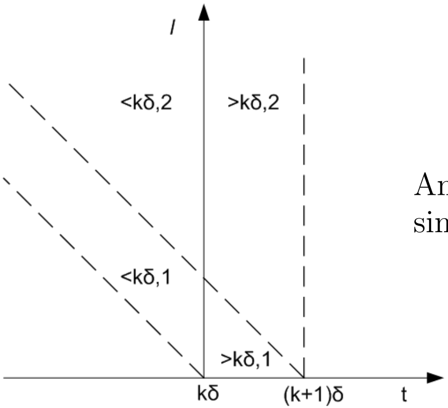
  where $\nu_0^{<0,2}$ is NOT a Lévy measure. Hint: should not expect id limit.
- Center and scale to get

  $$\frac{A^{<0,2}(0,\delta] - m(\delta)}{a(\delta)} \Rightarrow N(0,1).$$

The growth rate of $a(\delta)$ is given by

$$\begin{aligned} a^2(\delta) =& \mathbb{E}(F) \int_0^1 r\bar{G}_0(r)\mu_\delta(dr) \\ \sim& \mathbb{E}(F) \int_0^1 r\mu_\delta(dr) \\ =& (const)\frac{1}{\delta^{-1}\bar{F}_R(\delta^{-1})} \\ \sim& \mathbb{E}(F)\mathbb{E}(R)\frac{(\delta^{-1})^{(\alpha_R-1)}}{L_R(\delta^{-1})} \\ \to& \infty. \end{aligned}$$

6. Dependence across time slots.



Analyze cumulative input $A(k\delta, (k+1)\delta]$ similarly to $A(0, \delta]$ using shifted regions.

But what about dependence structure between

$$A(0, \delta] \text{ and } A(k\delta, (k+1)\delta]?$$

6.1. Dependence for $\delta \to 0$.

For any non-negative integer k, as $\delta \to 0$, in $\mathbb{R}^{k+1}$,

$$\frac{1}{a(\delta)}\left[\begin{pmatrix} A(0,\delta] \\ A(\delta,2\delta] \\ \vdots \\ A(k\delta,(k+1)\delta] \end{pmatrix} - \left\{2\int_0^1 v\bar{G}(v)\int_{r=v}^{\infty} r^{-1}\mu_\delta(dr)dv \right.\right.$$

$$\left.\left. - \int_0^1 \mathbb{E}(F)\bar{G}_0(r,\infty]\mu_\delta(dr)\right\}\begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}\right]$$

$$\Rightarrow \begin{pmatrix} X_\infty(0) \\ X_\infty(1) \\ \vdots \\ X_\infty(k) \end{pmatrix}$$

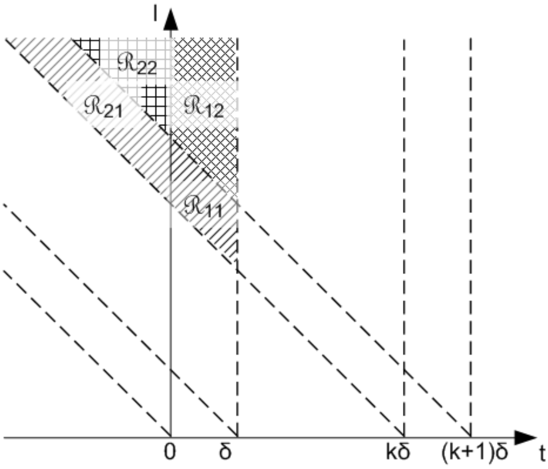
where the limiting sequence

$$\boldsymbol{X}_\infty = (X_\infty(k), -\infty < k < \infty\}$$

is Gaussian with

$$\mathrm{Corr}(X_\infty(0), X_\infty(k)) = 1.$$

Reason



- Four regions contribute to both $A(0,\delta]$ and $A(k\delta,(k+1)\delta]$.
- Region $\mathcal{R}_{22}$ contributes the Gaussian component to both $A(0,\delta]$ and $A(k\delta,(k+1)\delta]$.

- Common component

$$A^{\mathcal{R}_{22}}(0,\delta] = A^{\mathcal{R}_{22}}(k\delta,(k+1)\delta] = \sum_{(\Gamma_k,L_k,R_k,F_k)\in\mathcal{R}_{22}} R_k\delta.$$

- The scaling necessary to balance the contribution of $\mathcal{R}_{22}$ kills the asymptotically id contributions from the other 3 regions.

Conclude

The high frequency limit process obtained by letting $\delta \to 0$, has a degenerate dependence structure.

6.2. Dependence for fixed δ between slots.

For fixed $\delta > 0$, as $k \to \infty$,

$$\begin{aligned}\mathrm{Cov}(A(0,\delta], A(k\delta,(k+1)\delta]) \sim& (constant)\bar{G}_0(k)\\ \sim& (constant)k^{-(\alpha_F - 1)}L_F(k),\end{aligned}$$

where

$$(constant) \sim \int_0^\infty r^{2-\alpha_F}\mu_\delta(dr).$$

Thus the stationary sequence

$$\{A(k\delta,(k+1)\delta], -\infty < k < \infty\}$$

exhibits long range dependence. Note:

$$\int_0^\infty r^{2-\alpha_F}\mu_\delta(dr) \stackrel{\delta\to 0}{\to} \infty.$$

Method

We thus have

$$
\begin{aligned}
\mathrm{Cov}(A(0,\delta], & A(k\delta,(k+1)\delta]) \\
&= \mathrm{Cov}(A^{\mathcal{R}_{11}}(0,\delta], A^{\mathcal{R}_{11}}(k\delta,(k+1)\delta]) \\
&\quad + \mathrm{Cov}(A^{\mathcal{R}_{12}}(0,\delta], A^{\mathcal{R}_{12}}(k\delta,(k+1)\delta] \\
&\quad + \mathrm{Cov}(A^{\mathcal{R}_{22}}(0,\delta], A^{\mathcal{R}_{22}}(k\delta,(k+1)\delta]) \\
&\quad + \mathrm{Cov}(A^{\mathcal{R}_{21}}(0,\delta], A^{\mathcal{R}_{21}}(k\delta,(k+1)\delta]).
\end{aligned}
$$

The dominant term comes from the region $\mathcal{R}_{22}$; other terms of smaller order.

Reason

$$
\begin{aligned}
\mathrm{Cov}(A^{\mathcal{R}_{22}}(0,\delta], & A^{\mathcal{R}_{22}}(k\delta,(k+1)\delta]) \\
&= \mathrm{Var}(A^{\mathcal{R}_{22}}(0,\delta]) \\
&= \int_0^\infty r\bar{G}_0((k+1)r)\mu_\delta(dr).
\end{aligned}
$$

7. Summary.

Stylized Facts	Model
1. heavy tails	1. Built in
2. LRD across slots	2. Lag k cov $\sim c\bar{G}_0(k)$; δ fixed
3. Bursty	3. Traffic from regions $\mathcal{R}^{<0,1\cup>0,2\cup>0,1}$ id and compound Poisson with reg varying tails
4. Cumulative traffic per slot $\sim N(0,1)$	4. $\big(A(0,\delta] - (\text{centering}(\delta)\big)/a(\delta) \stackrel{d}{\approx} N(0,1)$
5. Dependence carried by β-traffic	5. Cov from id pieces smaller order than from Gaussian piece.

8. References

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